3.4 Discrete valued time series

Model description Most of the time series literature is concerned with continuous outcome. To construct models for time-correlated counts, one can use a 'state-space' approach. A state-space model has two components:

1. The 'state equation', which we implement using random effects.
2. The 'observation equation', which we take as the Poisson likelihood.

The state equation specifies that the state variable u_i follow a latent autoregressive process

$$u_i = au_{i-1} + e_i, \qquad e_i \sim N(0, \sigma^2).$$

There are two candidates for being our random effects: u_i and e_i.

★ If we take u_i to be the random effects, the model becomes separable.

If we instead choose e_i, the model does not become separable, because u_i depends on all of $e_1, \ldots, e_i$.) The ADMB-RE code for the separable model is

```
random_effects_vector u(1,n)
```

and

```
for (i=2;i<=n;i++)
  g += -log(sigma) -.5*square((u(i)-a*u(i-1))/sigma);
```

To exploit the separability structure, we need to place the above code in a `SEPARABLE_FUNCTION` section. The 'observation equation' for this model simply states that the observations y_i has a Poisson distribution with parameter $\lambda_i = \exp(\eta_i + u_i)$, where η_i is a linear predictor.

Results Zeger (1988) analyzed a time series of monthly numbers of poliomyelitis cases during the period 1970-1983 in the US. We make comparison to the performance of the Monte Carlo Newton-Raphson method of Kuk & Cheng (1999). Let y_i denote the number of polio cases in the i'th period. There are six covariates that account for trend and seasonal effects.

Estimates of hyper-parameters are shown in the following table.

	β_1	β_2	β_3	β_4	β_5	β_6	a	σ
ADMB-RE	0.242	-3.81	0.162	-0.482	0.413	-0.0109	0.627	0.538
Std. dev.	0.27	2.76	0.15	0.16	0.13	0.13	0.19	0.15
Kuk & Cheng (1999)	0.244	-3.82	0.162	-0.478	0.413	-0.0109	0.665	0.519

The standard deviation is large for several regression parameters. The ADMB-RE estimates (based on the Laplace approximation) are very similar to the exact maximum likelihood estimates as obtained with the method of Kuk & Cheng (1999).

Kuk & Cheng (1999) reported that the computation time for their method was 3120 seconds. The run time for ADMB-RE was 66 seconds.